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Duopsony mode Is with consistent

conjectural variations
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In this note we develop a consistent conjectural variation model that generalizes Bresnahan's (1981) results to a duopoly-duopsony setting.! This is the fIrst duopsony model in which flfIDS are constrained to have consistent conjectural variations, and two interesting results emerge! First, Bertrand conjectures occur when inputs are homo-
geneous and when input demand functions are horizontal. Second, the consistent con-
jectures equilibrium approaches Cournot when input demand functions are vertical or
when the flfID is a monopolist-monopsonist. This implies that firms will behave more
competitively in a duopsony setting as inputs and outputs become more homogeneous
and as input demand functions become relatively more elastic.

I. THE CONSISTENT CONJECTURAL geneous, thenpj = Pj and dpjldqj = dpjldqj, and if output is dif-
V ARIA TIONS MODEL ferentiated, thenpj * Pj and dpjldqj * dpj dqj'
In the input market, the same two flfIDS are the only flfIDS
Because flfIDS with monopsony power may also have mono- that employ input x. For simplicity, these flfIDS are assumed to poly power, a rather general duopsony-duopoly model with use just one inputS and face the same production function consistent conjectural variations will be considered.3 In terms when inputs are assumed to be homogeneous
of the output market, two flfIDS (1 and 2) are assumed to com- .= x. 2
pete in a single output market, where qj denotes the output of ql f(.) ( )
firm 1 or 2. Entry barriers are sufficiently high so that entry is There are many input suppliers who generate the following blocked.4 Because firms 1 and 2 are generally assumed to be input supply function
symmetrical, we shall focus on the behaviour of firm i and -
assume w. -w. (x. x.) (3) that flfID j is the other flfID when speaking about firm I I I' J'
i. There are a sufficiently large number of buyers who gener- where Wi denotes the price of Xi' Note that if the inputs are
ate the following inverse demand function for firm i: homogeneous, then Wj = Wj and dwjldXj = dWi/dXj, and if the
.= .(. .) (1) inputs are differentiated, then Wi * Wj and dwjldXj * dWi/dXj'
PI PI q.. qJ Firm i' s problem is then to choose x in order to maximize

where Pi denotes the price output. Note that if output is homo- profit, which is given by

I Although this modelling approach has its critics (Robson (1983), Lindh (1992?, its primary advantage is that, unlike game theory (Demsetz (1989), Fisher (1989), and Shepherd (1990?, it is easy to empirically test and leads to several interesting and testable implications. See Bresnahan (1989) and Schmalensee (1990) for a discussion of procedures to estimate output flfSt order conditions (or output conjectural variations) and Schroeter (1988) and Chang and Tremblay (1991) for procedures to estimate input first order conditions (or input conjectural variations).
2Schroeter (1988) and Chang and Tremblay (1991) developed oligopsony models using the conjectural variation approach but did not consider the case where conjectural variations are consistent. Several have developed consistent conjectural variation models for output markets. For example, see Bresnahan (1981), Laitner (1980), Ulph (1983), and Shaffer (1991). According to Pace and Gilley (1990), this model was first developed by Leontief (1936).
3Robinson (1934, p. 227) states that, 'The most important cases of monopsony will occur in connection with monopoly .'
4See Perry (1982) for a discussion of a consistent conjectures output model with free entry.
5We do not generally assume a fixed proportions technology, which would constrain input and output conjectures to be equal and,simply reproduce Bresnahan's (1981) results.
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Duopsony models with constant conjectural variations 9
nj = Pj(qj, qj)qj -Wj(Xj, Xj)Xj (4) same; v = vj = V2" Finally, a consistent conjectures equilibrium
T1h"blhf"'k(CCE) exists if all conjectural variations are consistent, denot-0 so ve t IS pro em, owever, Irm I must ma e some *" " " "
,bh' ,1'IIh" ed by v , and If each firm maximizes profits given the behav-assumption a out ow Its nva WI react to a c ange m Its ,f' "
, 'blIth'd1fi' ,dblour 0 ItS nv al"
own strategic vana es" n IS mo e, Irm I IS assume to e
concerned with j's input response to a change in xi" Thus, firm
i's first order (necessary) condition of profit maximization
requires: II, THE CONSISTENT CONJECTURES
onjloxj = [(opjloqj) (oqjloxj) + (opjloqj) (oqjloxj) (oxjlox)]qjEQUILIBRIUM
+p.(oq.lox.) -[Ow.lox. + (owlox.)(oxjox.)]x. -w. = 0 (5).I I ...J ..I This section will show how differing specifications of output
For notational convenience, this condition will be rewritten as demand, input supply and technology affect the CCE. First,
on. ox. = ! .' .' + .' .'v. .+ ..' -w.'x. -assume homogeneous inputs and outputs and that each firm
! .PI q. q., PJ qJ Iql P.q. I I faces the same linear output demand function, linear produc-w. vjX' -w. = 0 (6)J I I tion function, and linear input supply function. With these
where Pi' = opjloqj, Pj' = opjloqj' qj' = dqjldXj, qj' = oqjlxj, Wi' = assumptions, Equation 8 simplifies to:
OWjloxj, Wj' = owjloxj' and Vj = oxjOXj. The term v i represents .= -fp'('f -']
firm i's input conjectural variation (i,e", firm i's estimate of j's vJ q W
input reaction to a unit change in Xj)" p'(q')2(2+vj)-w'(2+v) (9)
Each firm's optimal input quantity cannot be determined ""' " .
"thkn.thalfth"1""IThe true consistent conjectural vanatlons are obtained by rec-WI out owIng e v ues 0 e conjectura vanatlons, n " " --* .,
od1"al"" "IIbdbeogruzmg that Vj -VJ' -v (given symmetry) and by solvIng
our me, conjectur vanatlons WI e assume to con- "* " .
,Bdfi"" "al"". " "f' Equation 9 for v , Note that under these conditions, the
slstent. y e mltlon, a conjectur vanatlon IS consistent I ItC'l'b"CCEACt1t'
.alth"alf""alth"l.b" oumot equi I num IS not a .oumo so u Ion
IS equ to e optlm response 0 Its nv at e equi I num , "' ,
dfidbth"Ffi'thoalthh' requires that Vj = VJ' = 0, but when Vj IS set to 0 m Equation 9,
e me y at conjecture. or Irm I IS equ s e c ange m al 1/2XJ" as indicated by j's reaction function, in response to a unit Vj eThq~ s -.
all1Bah'
h " IS case par e s resn an s (1981)Examp 1e 2because
c ange m Xj: , .it is the Bertrand equilibrium that is a CGE; v* = -I" Because
The consistent conjectures are calculated as follows. First,1""1f"
for simplicity "' It IS assumed that the market demand , function ISb0 tht he numerator an dt he denommator "0 fEqua tIon, "9the
negatively sloped and linear (at least locally), the Input supply"fu"dbI" &Bd'
"" .,. Input nctlon nee not e mear lor ertran conjec tures t0
function IS posItively sloped and linear (locally), and the pro-b"Gallh(1al)I' 't" "
d.fu..dr, ..ald" ,de consistent. ener y owever, oc mean y IS required
uctlon nctlon IS qua atlc, pOSItive v ue , IncreasIng, an "d&th1fth"f"
""" "" 6" m or er lor e s ope 0 e reaction unction to bea cons tant
concave fi.,over pOSItive Input quantities, At optimal values,d&. 7F"alltthhv*-1th"t
fidd" " ..d.dbeallan lor Uniqueness, m y, no e at w en --, e mpu
mn I s Irst or er con Itlon IS an I entity an can tot y " , "
dot"' "db" market operates efficiently sInce the value of the margInal
I lerentlate to 0 taln:pr oduct & lor Input "x equ als the pnce '0 fInput "x from the fiIrst
r n.'q."q.+P.'q.'q.' +P.'q.' v,q.' +P.'q.'q.' +P.q" -w.' order condition
IJ'. I .I. I J J I I I .
-Wj'Vj -Wj']dxj + fpj' qj" Vjqj + Pj' qj' qj' -Wj']OXj = 0 (7) Next, we relax the linear technology constraint by allowing
h,,:\ '/:\d" :\ '
/:\B" the production function to be concave. With this added
were
,
q
.-.I .= Qq.ox, anqJ .= -J Qq.ox,J' YrearrangIngterms, assumption Equation 8 becomes
Equation 7 can be expressed as
-(p'q"v*q + P'q'q' -w'oxjOXj =(p' "v + ' " ')v* =
-j qj iqj Pj qj qj -Wj 8 p'q"q + pq" + p'(q')2(2 + v*) -w'(2 + v*) (10)
", "", , , , " " , ()
pj qj qi+Pi qi qi +Pj qj viqi +Pi qrtJi +Piqi -Wi -Wj vrwi Again, Equation 10 shows that as the production function
This equation has three interpretations: (1) it equals the slope become linear (q" = 0), the CCE approaches the Bertrand
of firm i's input reaction function; (2) when evaluated at op- equilibrium" Furthermore, by solving for v* one can see that
timal values, it equals the true equilibrium change in Xi in as the marginal product and input demand approach the ver-response to a unit change in Xj and (3) it equals fmnj's consis- tical (q" approaches 00), v* approaches 0 and the input market
tent input conjectural variation when solved for Vj. Note that CCE approaches Cournot,8 This parallels Bresnahan's (1981)
because of symmetry (i,e" fIrmS 1 and 2 are assumed to face Example 3 and shows that the CCE departs from Bertrand and
the same demand and technological conditions and behave the becomes less competitive as input demand functions become
same), each fInn's consistent conjectural variation will be the steeper.
~ese assumptions simplify the analysis by ensuring that the consistent conjectural variations will be constants" See Bresnahan (1981) for
further discussion"
7See Bresnahan (1981) for further discussion of this point in relation to output markets and the Appendix for a more complete derivation of the
consistent conjectural variation and a discussion of uniqueness and the second order condition.

10 Yang-Ming Chang and ~ J. Tremblay

Finally, we will analyse the CCE when inputs and outputs most likely when inputs are homogeneous and when input are differentiated. For simplicity, we assume a linear produc- demand functions are horizontal, and Cournot conjectures are
tion function. In this case, Equation 8 becomes most likely when input demand functions are vertical or when
- (Pj' qj' qi' -Wj') the fInn is a monopolist-monopsonist.
v*=
2Pi'(qi')2 + Pj'qj'v*qi' -2Wi' -Wj'v* (11)

Again, the CCE is Bertrand only in the polar case where out- ACKNOWLEDGEMENTS
puts and inputs are perfect substitutes (i.e., Pi' = Pj' and Wi' =
Wj'). At the other pole, Equation 11 implies that when outputs Comments from Chi-Chur Chao, Carlos Martins, Art and inputs are not substitutable at all (Pj' and Wj' approach 0), 0' Sullivan and Carol Horton Tremblay are gratefully then v* approaches 0 and the model generates a monopoly- acknowledged. Any remaining errors are the responsibility of
monopsony solution with just one fInn. the authors.
The results of the model have several interesting implica-
tions. First, they parallel those of Bresnahan (1981), who
found that under similar conditions the output CCE becomes REFERENCES
less competitive as rival outputs become less substitutable and
as marginal cost functions become steeper. In addition, this Bresnahan, T. F. (1981) Duopoly Models with Consistent
analysis helps to establish situations where Bertrand and Conjectures, American Economic Review, 71, 934--45.
Cournot input conjectures are sensible under the criteria estab- Bresnahan, T. F. (1983) Duopoly Models with Consistent
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slope input demand functions approaches infinity, Cournot Bresnahan, T. F. (1989) Empirical Studies of Industries with Market
conjectures are sensible because it is appropriate for a fInn to Power, in Handbook .of Industrial Organization (R.
ak th t t .. bl d th t. I,. t Schmalensee and R. D. WillIg, eds), North-Holland, New York,
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quantities will remain fixed (a 0 input quantity conjectural Chang Y.-.an M d V..rem J T blay (1991) 01.Igopsony 101.Igopo 1y
vanatlon). AlternatIvely, If mputs are homogeneous and mput Power and Factor Market Performance, Managerial and
demand functions are horizontal, Bertrand conjectures are Decision Economics, 12,405-9.
sensible because it would then be appropriate for the firm to Demsetz, H. (1989) Industrial Organization at Age Sixty, UCLA
make input price the strategic variable and assume that rivals' working paper, February 9,1989.
input prices will remain fixed (a 0 input price conjectural vari- Fisher, F. (1989) Games Econontists Play: A Noncooperative View,
ation). Finally, if a firm is a monopolist and a monopsonist Rand Journal of Economics, 20, 113-24.
(i.e., the fInn's output and inputs have no substitutes), then a Fisher, I. (1989) Cournot and Mathematical Econontics, Quarterly
Cournot conjecture is sensible because it generates the Journal of Economics, 12, 117-38.
monopoly-monopsony solution. Kamien, M. I. and Schwartz, N. I. (1983) Conjectural Variations,
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III. CONCLUSION University Press, Princeton, USA.
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In this note we develop a consistent conjectures model for a Leo. f W S k lbe M 1. ti. C t.t" ,
ntie , ., tac e rg on onopo IS c ompe I Ion, Journa I 0if
duopsony that leads to several mterestIng and testable lmphca- Political Economy, August 1936,554-9.
tions. First,.t~e model pre~icts that an inp~t market will be Lindh, T. (1992) The Inconsistency of Consistent Conjectures,
less competItIve when the mputs that are hIred by each firm Journal of Economic Behavior and Organization, 18, 69-90.
become more differentiated and when the input demand func- Pace, R. K. and Gilley, W. (1990) A Note on Consistent Conjectures:
tions become relatively more inelastic. This result parallels The Leontief Legacy. Southern Economic Journal, 56,788-92.
Bresnahan's (1981) for a duopoly setting and is a rather 'intu- Perry, M. K. (1982) Oligopoly and Consistent Conjectural
itive theory of competition,9 because one might expect prod- Variations, Bell Journal of Economics, 13, 197-205.
uct differentiation and the structure of input demand functions Robinson, J. (1934) The Economics of Imperfect Competition,
to affect input market performance in this way. In addition, Macntillan, London, UK.
our results clarify the conditions under which Bertrand and Robson, A. (1983) Existence of Consistent Conjectures: Comment,
Cournot conjectures might occur. Bertrand conjectures are American Economic Review, 73,454-6.

81n this case, the second order condition of profit maxintization requires that:
q"(p'q + p) + p'(q')2(2 + VJ -w'(2 + vJ < O. This condition is satisfied if vi < 2 [which is reasonable since a fIrm's conjectural variation is likely to lie between Bertrand (Vi = -1) and car-tel (Vi = 1)], the output demand function is negatively sloped (p' < 0), the input supply function is positively sloped (w'>O), and p' q + P > O. Note that this last term can be rewritten as p[1 -(If)] [where n = (dqldp (plq) > 0] and will be positive if the equilibrium price is greater than zero and the firm operates in the elastic region of demand.
~s is the phrase used by Bresnahan (1981, p. 943) to describe the implications of his duopoly with consistent conjectures.

Duopsony models with constant conjectural variations 11

Schrnalensee, R. (1990) Empirical Studies of Rivalrous Behaviour, in which simplifies to
(G. Bonanno and D. Brandolini, eds), Industrial Structure in the -1 (AI)
New Industrial Economics, Cl~endon ~ess, Oxford, u~. ..Vj = -(~)
Shaffer, S. (1991) Consistent Conjectures m a Value-Maxlmlzmg ..
Duopoly, Southern Economic Journal, 57,993-1009. Setting vi = Vj = v (by symmetry), EquatIon Al can be rewnt-
Shepherd, W. G. (1990) Publishing Performance in Industrial ten as
Organization: A Comment, Review of Industrial Organization, 2
5,139-46. V +2v+ 1 =0. (A2)
Ulph, D. (1983) Rational Conjectu~s in the .Th~ory of Oligopoly, The roots of this quadratic equation produce single consistent
International Journal of Industrial Organization, 131-54. .

rural . t.
conJec vana Ion equ al t0 -.1
Under the assumptions given above, the sac of profit
maximization is satisfied. To see this, note that profit maxim-
APPENDIX ization requires that

This appendix demonstrates that there is a unique consistent p'(q')2(2 + VJ -w'(2 + vJ < O. (A3)
conjecture that satisfies the second order sufficient (necessary) This condition is satisfied when Vi = -1 and the output condition (SaC) when inputs and outputs are homogeneous demand and input supply functions are regular (i.e., p' < 0 and and when output demand, input supply, and the production w' > 0, respectively).
function are linear. Uniqueness can be established as follows.
Recall Equation 9
_fp'(q')2 -w1

y-j -p'(q')2(2 + vJ -w'(2 = vJ